

## TECHNICAL APPENDIX TO “WHAT IS A BARRIER TO ENTRY?”

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First, we argue formally that scale economies are ancillary, antitrust barriers to entry. To do so, we present a simple model in which (1) scale economies do not delay entry on their own, (2) brand loyalty delays entry on its own, and (3) brand loyalty delays entry even longer in the presence of scale economies.

Consider a one-shot entry game. A potential entrant first chooses whether or not to enter a market. If it chooses not to enter, the sole incumbent acts as a monopolist. If it chooses to enter, the entrant and incumbent play a Cournot duopoly game. The entrant and incumbent both have the same cost function  $C(q) = cq + f$ , where  $c$  is marginal cost, and  $f$  is fixed cost (the simplest expression of scale economies). Note that incumbent and entrant both have to bear the fixed cost  $f$ . Therefore,  $f$  is certainly not an economic barrier to entry in this model. The incumbent's inverse demand function is given by  $P(q) = 1 - Q$ , where  $Q$  is the total quantity produced by the industry, that is,  $Q = q_I + q_E$  if the potential entrant chooses to enter the market, and  $Q = q_I$  otherwise, where  $q_I$  and  $q_E$  are the incumbent's and entrant's quantity choices, respectively. The potential entrant's inverse demand function, if it chooses to enter the market, is given by  $P(q) = 1 - Q - \lambda$ , where  $\lambda$  is a measure of consumers' loyalty to the incumbent's brand. Note that if  $\lambda$  deters entry, then it is an economic barrier to entry, since in this model it is a cost to the entrant but not to the incumbent.

**Case 1.**  $\lambda = 0$  and  $f > 0$

In the subgame that follows entry, the incumbent and entrant's maximization problem is

$$(1) \quad \max_{q_i} (1 - Q)q_i - cq_i - f$$

The equilibrium quantity choices of the incumbent and entrant are

$$(2) \quad q_I = q_E = \frac{1 - c}{3}$$

Therefore, the incumbent and entrant's equilibrium profits are given by

$$(3) \quad \pi_I = \pi_E = \frac{(1 - c)^2}{9} - f$$

Hence, the potential entrant chooses to enter if and only if

$$(4) \quad f \leq \frac{(1 - c)^2}{9}$$

Note that if the marginal and fixed costs are small enough, inequality (4) is satisfied, so that the fixed cost never deters entry, in the absence of brand loyalty. In other words, for the parameter ranges defined by (4), scale economies are not primary barriers to entry.  $\square$

**Case 2.**  $\lambda > 0$  and  $f = 0$

In this case, the entrant's maximization problem is

$$(5) \quad \max_{q_E} (1-Q-\lambda)q_E - cq_E$$

The first order condition yields

$$(6) \quad q_E = \frac{1-q_I-c-\lambda}{2}$$

The incumbent's maximization problem is

$$(7) \quad \max_{q_E} (1-Q)q_E - cq_E$$

Here, the first order condition yields

$$(8) \quad q_I = \frac{1-q_E-c}{2}$$

Solving (7) and (8) simultaneously yields

$$(9) \quad q_I = \frac{1-c+\lambda}{3} \quad \text{and} \\ q_E = \frac{1-c-2\lambda}{3}$$

Therefore, the potential entrant's profits, if it chooses to enter the market, are given by

$$(10) \quad \pi_E = \frac{(1-c-2\lambda)^2}{9}$$

The equation  $\pi_E = 0$  has the following root:

$$(11) \quad \lambda_1 = \frac{1-c}{2}$$

Therefore, the potential entrant chooses to enter if and only if

$$(12) \quad \lambda < \frac{1-c}{2}$$

where the quantity on the right hand side is the monopoly output. When brand loyalty is large enough, inequality (12) is not satisfied, and so entry is deterred, even in the absence of scale economies. In this case, brand loyalty is a primary, economic barrier to entry.  $\square$

**Case 3.**  $\lambda > 0$  and  $f > 0$

In this case, the entrant's maximization problem is

$$(13) \quad \max_{q_E} (1-Q-\lambda)q_E - cq_E - f$$

And the incumbent's maximization problem is

$$(14) \quad \max_{q_E} (1-Q)q_E - cq_E - f$$

The solutions to (13) and (14) are the same as the solutions to (5) and (7):

$$(15) \quad q_I = \frac{1-c+\lambda}{3} \quad \text{and} \\ q_E = \frac{1-c-2\lambda}{3}$$

Therefore, the potential entrant's profits, if it chooses to enter the market, are given by

$$(16) \quad \pi_E = \frac{(1-c-2\lambda)^2}{9} - f$$

The equation  $\pi_E = 0$  now has the following two roots:

$$(17) \quad \lambda_{1,2} = \frac{1-c \pm 3\sqrt{f}}{2}$$

Therefore, the potential entrant chooses to enter if and only if

$$(18) \quad \lambda < \frac{1-c-3\sqrt{f}}{2} < \frac{1-c}{2}$$

Hence brand loyalty deters entry for a larger range of parameters with scale economies than without them.

Can this imply that brand loyalty delays entry longer with scale economies than without them? The model does not have an explicit time dimension, but we can nevertheless address the issue of entry delay indirectly by considering how the model's parameters might change over time. Suppose that technological innovation in input markets will continuously reduce the industry's marginal cost  $c$  for all of its participants. Then, entry would eventually take place, all else approximately constant, for as  $c$  decreases, the inequalities in (18) are more likely to be satisfied. But entry would take place later with scale economies than without them, since the first inequality in (18) is stricter than the second.

Does the additional delay in entry occasioned by scale economies necessarily reduce social welfare? For an important class of demand functions (including linear demand), social welfare under Cournot competition is higher than social welfare under monopoly, because the profit loss incurred by the incumbent is not large enough to offset the price reduction that benefits consumers. In these cases, scale economies are ancillary, antitrust barriers to entry, since they delay entry by reinforcing the entry deterrent effects of brand loyalty, and thereby reduce social welfare.  $\square$

Second, we argue formally that sunk costs are ancillary, antitrust barriers to entry also. To do so, we present a simple model in which (1) sunk costs do not delay entry in the absence of uncertainty, (2) uncertainty does not delay entry in the absence of sunk costs, but (3) uncertainty and sunk costs combine to delay entry.

Consider a two-period entry deterrence model in which a prospective entrant is uncertain about the incumbent's type. The incumbent is either aggressive, with probability  $\alpha$ , or weak, with probability  $1-\alpha$ . The aggressive incumbent never accommodates. In period 1, the potential entrant chooses whether or not to enter, not knowing the incumbent's type. If the potential entrant enters, the weak incumbent chooses whether or not to accommodate. If the incumbent does not accommodate, its payoff is  $0 + \delta\pi^m$ , where  $\delta$  is the discount factor, and the entrant's payoff is  $-\sigma$ , where  $\sigma$  is a measure of the extent to which the capital costs of entering the industry are sunk. If the weak incumbent accommodates, the weak incumbent and entrant both get the Cournot payoff,  $\pi^c$ , in each of the two periods, for a total payoff of  $(1+\delta)\pi^c$ .

If the potential entrant does not enter in period 1, it chooses whether or not to enter in period 2. At the end of period 1, just before period 2, the entrant learns the incumbent's type (perhaps because it has had time to observe the incumbent's reaction to other entrants). If the potential entrant does not enter in either period, its payoff is 0, and the incumbent's payoff is  $\pi^m(1+\delta)$ , where  $\pi^m$  is the monopoly profit. If the incumbent does not accommodate in period 2, then its payoff is  $\pi^m$  and the

entrant's payoff is  $-\delta\sigma$ . If the weak incumbent accommodates in period 2, then its payoff is  $\pi^m + \pi^c\delta$  and the entrant's payoff is  $\pi^c\delta$ . Notice that if  $\delta\pi^m < (1+\delta)\pi^c$ , the incumbent never accommodates, and hence the potential entrant never enters if it has to incur any positive sunk entry cost. Henceforth, we assume that  $\delta\pi^m < (1+\delta)\pi^c$ .

**Case 1.**  $\alpha \in \{0,1\}$  and  $\sigma > 0$

Suppose  $\alpha = 0$ . By backwards induction, the incumbent accommodates in both periods, and hence the entrant enters in period 1, regardless of  $\sigma$ . Now suppose  $\alpha = 1$ . In this case, the entrant knows that the incumbent never accommodates, and therefore it never enters, whether  $\sigma$  is small or large. Thus, large sunk costs do not delay entry, or do not cause additional entry delay, in the absence of uncertainty. In other words, sunk costs are not primary barriers to entry.  $\square$

**Case 2.**  $\alpha \in (0,1)$  and  $\sigma = 0$

By backwards induction, the weak incumbent accommodates in both periods. Therefore, the potential entrant enters in period 2 if it has learned at the end of period 1 that the incumbent is weak, but does not enter if it has learned that the incumbent is aggressive. Now, the potential entrant's expected payoff from not entering in period 1 is  $(1-\alpha)\delta\pi^c$  (which is a measure of the lost option value of entering), while its expected payoff from entering in period 1 is  $(1-\alpha)(1+\delta)\pi^c$ . Thus, the potential entrant always enters in period 1. Thus, uncertainty never deters entry, in the absence of sunk entry costs. In other

words, uncertainty is not a primary barrier to entry either.  $\square$

**Case 3.**  $\alpha \in (0,1)$  and  $\sigma > 0$

By backward induction, we find, once again, that the weak incumbent accommodates in both periods, and therefore, the potential entrant enters in period 2 if it learns that the incumbent is weak, but does not enter if it learns that the incumbent is aggressive. The potential entrant's expected payoff from not entering in period 1 is still  $(1-\alpha)\delta\pi^c$ , but now its expected payoff from entering in period 1 is  $\alpha(-\sigma) + (1-\alpha)(1+\delta)\pi^c$ . Therefore, the potential entrant does not enter in period 1 if and only if

$$(19) \quad \sigma > \frac{1-\alpha}{\alpha} \pi^c$$

Thus, large sunk costs (high  $\sigma$ ) and uncertainty ( $\alpha$  not too small) can combine to delay entry until the realization of uncertainty. For an important class of demand functions, efficient entry is in advance of the realization of uncertainty. Hence, sunk costs and uncertainty are ancillary, antitrust barriers to entry that combine, and reinforce each other, to produce a primary, antitrust barrier to entry.  $\square$